(b) To test hypothesis (i), try at least three different lengths of chains with the controls being the same amplitude, same child, and same effort put into the push. To test hypothesis (ii), try at least three different amplitudes with the same length of swing, same child, and same effort put into the push. To test hypothesis (iii), try pushing at least three different children of different masses with the controls being the same length, same amplitude and the same push. For testing (iv), try different pushes (gentle, medium, and hard) with the controls being the same length, the same amplitude, and the same child.

9. Graph A is the correct graph because as the temperature increases, the speed of sound increases.

Making Connections

10. Answers will vary. An example could be a guitar with an amplifier to improve the sound.
11. Acoustics are the qualities of an auditorium that determine how well sound is heard. An auditorium with good acoustics will cause the sound to reflect off surfaces to ensure that everyone in the auditorium will hear the sound. An important acoustics property of an auditorium is its reverberation time (the time required for the sound intensity to drop to its original value).
12. Look for answers that involve feeling or seeing the vibrations that produce sound waves.

CHAPTER 6 VIBRATIONS AND WAVES

Try This Activity: Wave Action

(Page 195)

This experiment is a good way to demonstrate some examples of waves. Students will note how the particles in a wave move.

6.1 VIBRATIONS

PRACTICE

(Page 198)

Understanding Concepts

1. (a) The tree undergoes transverse vibrations.
   (b) The needle undergoes longitudinal vibrations.

2. (a)
$$T = \frac{\text{total time}}{\text{number of cycles}}$$
$$= \frac{15\text{ s}}{25\text{ cycles}}$$
$$T = 0.60\text{ s}$$

   (b)
$$T = \frac{\text{total time}}{\text{number of cycles}}$$
$$= \frac{60\text{ s}}{15\text{ cycles}}$$
$$T = 4.0\text{ s}$$

   (c)
$$T = \frac{\text{total time}}{\text{number of cycles}} \quad 33699$$
$$= \frac{60\text{ s}}{2450\text{ cycles}}$$
$$T = 2.4 \times 10^{-2}\text{ s}$$

3. $f = \dfrac{1}{T}$

$= \dfrac{1}{\frac{1}{80}\text{ s}}$

$f = 80\text{ Hz}$

4. $f = \dfrac{\text{number of cycles}}{\text{total time}}$

$= \dfrac{20\text{ cycles}}{25\text{ s}}$

$f = 0.80\text{ Hz}$

$T = \dfrac{1}{f}$ or $T = \dfrac{25\text{ s}}{20\text{ cycles}}$

$= \dfrac{1}{0.80\text{ Hz}}$ $\quad T = 1.2\text{ s}$

$T = 1.2\text{ s}$

5. $f = \dfrac{2.40 \times 10^4\text{ cycles}}{60\text{ s}}$

$f = 4.00 \times 10^2\text{ Hz}$

$T = \dfrac{1}{400\text{ Hz}}$

$T = 2.50 \times 10^{-3}\text{ s}$

6. (a) $f = \dfrac{\text{number of cycles}}{\text{total time}}$

$= \dfrac{88\text{ cycles}}{0.20\text{ s}}$

$f = 4.4 \times 10^2\text{ Hz}$

(b) $f = \dfrac{3600\text{ cycles}}{60\text{ s}}$

$f = 60\text{ Hz}$

(c) $f = \dfrac{4.5 \times 10^3\text{ cycles}}{60\text{ s}}$

$f = 75\text{ Hz}$

7. $T = \dfrac{163.8\text{ d}}{6\text{ cycles}}$

$T = 27.30\text{ d}$

8. As you walk, the movement of your right leg is in phase with your left arm and out of phase with your right arm.

Investigation 6.1.1 The Pendulum

(Pages 199–201)

Purpose

The purpose of this investigation is to determine the factor(s) that influence the frequency of a swing of a pendulum.

Question

What are the relationships between the frequency of a simple pendulum and its mass, amplitude, and length?

Hypothesis/Prediction

(a) It is expected that the frequency of a pendulum is determined solely by its length. Watching people on playground swings, it would appear that a person's mass does not influence the rate at which the apparatus swings back and forth. The amplitude also does not seem to play a role. As a person allows the swing to come to rest with the amplitude gradually decreasing, the frequency does not appear to change.

Design

A simple pendulum was constructed and allowed to swing, measuring the frequency in each trial. While two of the factors (mass, amplitude, and length) are controlled, the third was varied to determine its influence, if any, on the frequency. Once the factor was identified, a quantitative analysis of its influence was sought.

Materials

- utility stand
- clamp
- test-tube clamp
- split rubber stopper
- string
- stopwatch
- metre stick
- metal masses (50 g, 100 g, 200 g)

Procedure

1. A pendulum was constructed using a string with a mass suspended from its end. The other end of the string was secured in a split rubber stopper held in place on a retort stand with a test-tube clamp.
2. The pendulum was adjusted in each trial so that its length was the distance from the point where the string passes through the rubber stopper to the middle of the suspended metal mass.
3. In the first set of trials the pendulum's length was set at 100.0 cm using a 200.0-g mass. The mass was pulled to the side, keeping the string tight, to an amplitude of 10.0 cm and then released. The time for the pendulum to complete 20 full cycles was measured. This was repeated two more times and the average value of the three attempts was recorded in **Table 1**.
4. Step 3 was repeated using an amplitude of 20.0 cm and again with an amplitude of 30.0 cm.
5. In a second set of trials, the pendulum's length remained at 100.0 cm and the amplitude at 10.0 cm. The mass was varied, suspending masses of 50.0 g, 100.0 g, and 200.0 g in turn, each time measuring and recording the average the time taken for 20 full cycles. The results were recorded in Table 1.
6. In a third set of trials, the pendulum's mass remained constant at 100.0 g and the amplitude at 10.0 cm. The length varied using values of 100.0 cm, 80.0 cm, 60.0 cm, 40.0 cm, and 20.0 cm. The average time in each case was recorded in Table 1.

Observations

In each trial the pendulum was seen to swing back and forth at a constant rate. The time to complete 20 full cycles was measured three times for every set of values of mass, amplitude, and length. The average of these times was recorded in Table 1. When these times are divided by 20, the period of the pendulum's swing was found. Its frequency (also recorded) was calculated using the expression $f = \frac{1}{T}$. Using the first measurements in Table 1 as an example:

average time for 20 full cycles: 39.5 s

period of swing: $T = \frac{3.95\text{ s}}{20} = 1.98\text{ s}$

frequency of swing: $f = \frac{1}{T} = \frac{1}{1.98\text{ s}} = 0.506\text{ Hz}$

It was noted that in the first two sets of trials the pendulum's frequency did not appear to vary with either changing amplitude or changing mass. In the third set of trials where the mass and amplitude were fixed and the pendulum's length was allowed to vary, there was a noticeable difference in its frequency at various lengths. The pendulum appeared to swing with a smaller period (greater frequency), as it became increasingly shorter.

Table 1 Factors Affecting the Frequency of a Pendulum

Length (cm)	Mass (g)	Amplitude (cm)	Average Time for 20 Full Cycles (s)	Frequency (Hz)
100.0	200.0	10.0	39.5	0.506
100.0	200.0	20.0	40.5	0.494
100.0	200.0	30.0	40.0	0.500
100.0	50.0	10.0	40.0	0.500
100.0	100.0	10.0	40.5	0.494
100.0	200.0	10.0	39.5	0.506
100.0	100.0	10.0	40.0	0.500
80.0	100.0	10.0	35.9	0.557
60.0	100.0	10.0	31.1	0.643
40.0	100.0	10.0	25.4	0.788
20.0	100.0	10.0	17.6	1.11

Analysis

(b) Graphs of Frequency – Amplitude, (**Figure 1**); Frequency – Mass, (**Figure 2**); and Frequency – Length, (**Figure 3**) are plotted on the next page.

(i)

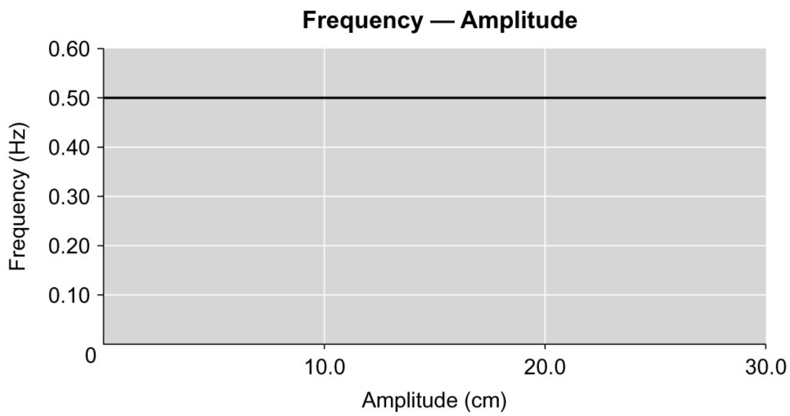


Figure 1
Frequency and Amplitude

(ii)

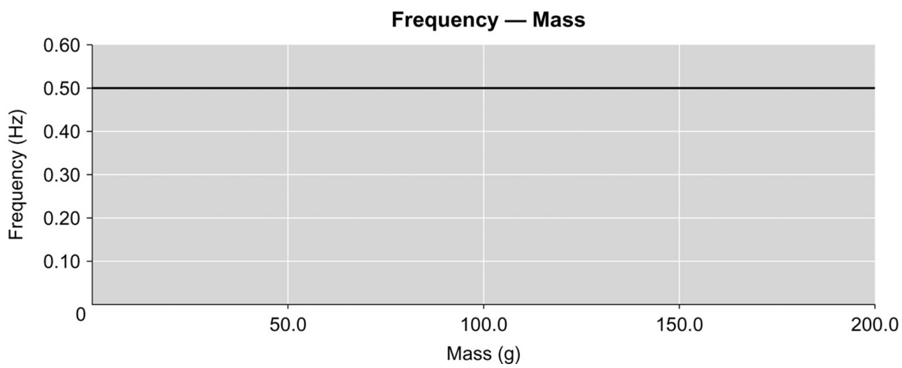


Figure 2
Frequency and Mass

(iii)

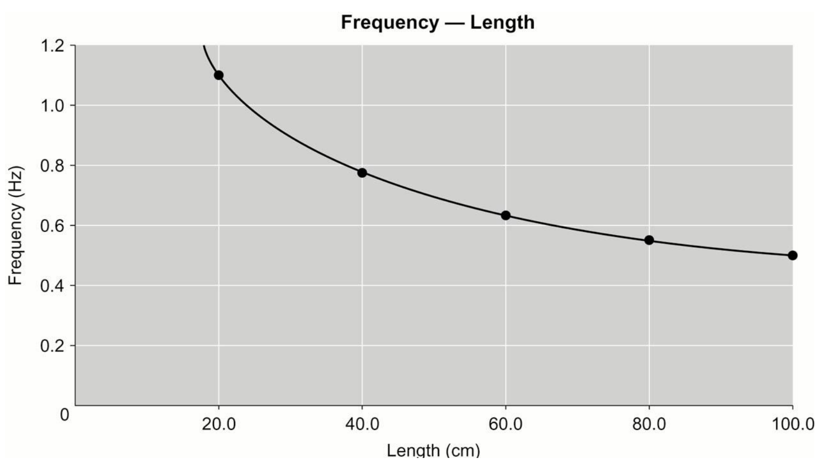


Figure 3
Frequency and Length

(c) The results indicate that the factor that determines the frequency of a pendulum is its length. The mass and amplitude did not play a role. The frequency decreased as the length increased.

(d) The periods of the pendula in the third set of trials were determined and recorded in **Table 2**. The period was calculated in each case using $T = \frac{1}{f}$. For example, in the first set of results for the trials where length was the variable:

$$f = 0.500 \text{ Hz}$$
$$T = \frac{1}{f}$$
$$= \frac{1}{0.500 \text{ Hz}}$$
$$T = 2.00 \text{ s}$$

Table 2 Period and Length of Pendula, Constant Mass and Amplitude

Length (cm)	Period (s)
100.0	2.00
80.0	1.80
60.0	1.56
40.0	1.27
20.0	0.90

The results of Table 2 are plotted in **Figure 4**.

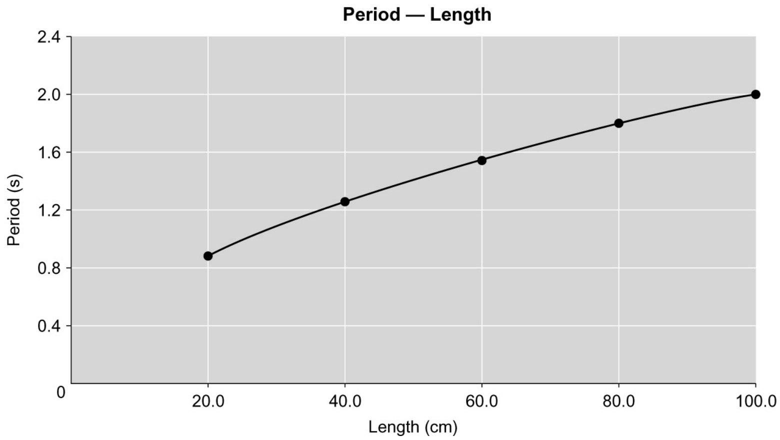


Figure 4
Period and Length

The nature of the relationship between period and length is illustrated by the shape of Figure 4. As the length of the pendulum increases, the period of its swing increases.

Table 3 is a summary of period and the square root of the pendulum's length.

Table 3 Period and Square Root of Length

Period (s)	Square root of length ($cm^{\frac{1}{2}}$)
2.00	10.0
1.80	8.94
1.56	7.75
1.27	6.32
0.90	4.47

The data in Table 3 are plotted in **Figure 5**.

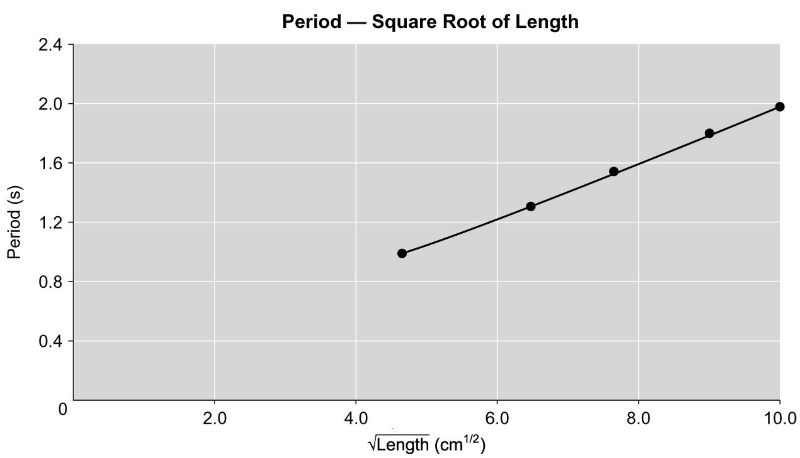


Figure 5
Period and Square Root of Length

It is apparent from this graph that the period of a pendulum's swing is directly proportional to the square root of its length.

Evaluation

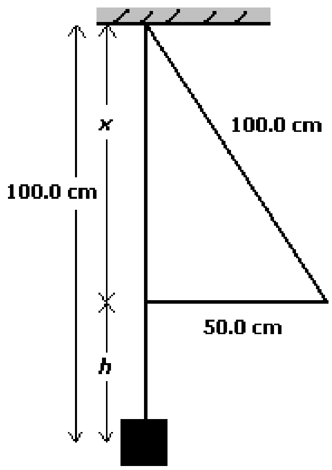
(e) The predictions made in the hypothesis were correct. The only factor determining the frequency of a pendulum is its length. From the mathematical analysis, it can be concluded that the frequency of a pendulum is inversely proportional to the square root of its length.

(f) There were a number of experimental errors associated with this investigation. The largest source of error involved obtaining accurate values for the time. The observer had to use best judgement when deciding when the pendulum had reached the end of its swing. Hand-eye coordination also played a role when starting the stopwatch at the beginning of the swing. The retort stand was seen to sway slightly from side to side as the pendulum swung back and forth. This may have had an effect on the results, perhaps resulting in times that were slightly greater than theoretical values. The pendulum did not always make uniform swings over the 20 full cycles. In some cases the plane of the pendulum's swing slowly rotated about the top of the pendulum. It is expected that this would have a negligible effect on the pendulum's frequency.

To improve the experimental design, a more rigid support stand could be used. Improvements to timing could be made by starting the pendulum swinging and then begin timing when the pendulum reaches its highest point on some subsequent swing rather than beginning when the pendulum is first released.

Synthesis

(g) A pendulum that is 100.0 cm long has a mass of 100.0 g suspended from its end. It is pulled to the side, giving it an amplitude of 50.0 cm. As the pendulum is displaced sideways, it becomes raised vertically giving it some gravitational potential energy. Upon its release, the gravitational potential energy it had gained is converted into kinetic energy as it swings downward. Its speed at the bottom of the swing can be calculated as follows.



Using Pythagorus:

$x = ((100.0 \text{ cm})^2 - (50.0 \text{ cm})^2)^{\frac{1}{2}}$
$x = 86.6$ cm

$h = 100.0 \text{ cm} - 86.6 \text{ cm}$
$h = 13.4$ cm (0.134 m)

Gain in gravitational potential energy when pendulum is raised:

$m = 100.0$ g (0.1000 kg)
$g = 9.8$ N/kg
$h = 0.134$ m

$E_g = mgh$
$= 0.1000 \text{ kg}(9.8 \text{ N/kg})(0.134 \text{ m})$
$E_g = 0.13$ J

Gain in kinetic energy when pendulum swings through the lowest point: $E_K = 0.131$ J

$m = 0.1000$ kg
$E_{ke} = 0.131$ J

$$v = \left(2\left(\frac{E_k}{m}\right)\right)^{\frac{1}{2}}$$
$$= \left(2\left(\frac{0.131 \text{ J}}{0.1000 \text{ kg}}\right)\right)^{\frac{1}{2}}$$
$$v = 1.6 \text{ m/s}$$

The speed of the pendulum at its lowest point is 1.6 m/s.

Section 6.1 Questions

(Pages 201–202)

Understanding Concepts

1. (a) transverse vibration
   (b) transverse vibration
   (c) longitudinal vibration
2. (a) The pendulum undergoes transverse vibrations.
   (b) The amplitude of vibration is 8.5 cm.
   (c) In one cycle, the mass moves 17 cm × 2 = 34 cm. In five cycles, the mass moves a total distance of 5 × 34 cm = 170 cm (assuming the amplitude remains constant).
3. (a) $f = \frac{\text{number of cycles}}{\text{total time}} = \frac{1800 \text{ pictures}}{60 \text{ s}}$, $f = 30$ Hz

   $T = \frac{\text{total time}}{\text{number of cycles}} = \frac{60 \text{ s}}{1800 \text{ pictures}}$, $T = 3.3 \times 10^{-2}$ s

(b) $f = \dfrac{1800 \text{ beats}}{20.0 \text{ s}}$ $\qquad$ $T = \dfrac{20.0 \text{ s}}{1800 \text{ beats}}$

$f = 90 \text{ Hz}$ $\qquad$ $T = 1.1 \times 10^{-2} \text{ s}$

(c) $f_1 = \dfrac{460 \text{ cycles}}{60 \text{ s}}$ $\qquad$ $T_1 = \dfrac{60 \text{ s}}{460 \text{ cycles}}$

$f_1 = 7.7 \text{ Hz}$ $\qquad$ $T_1 = 0.13 \text{ s}$

$f_2 = \dfrac{640 \text{ cycles}}{60 \text{ s}}$ $\qquad$ $T_2 = \dfrac{60 \text{ s}}{640 \text{ cycles}}$

$f_2 = 10.7 \text{ Hz or } 11 \text{ Hz}$ $\qquad$ $T_2 = 9.4 \times 10^{-2} \text{ s}$

(d) $f = \dfrac{60 \text{ cycles}}{60 \text{ s}}$ $\qquad$ $T = \dfrac{60 \text{ s}}{60 \text{ cycles}}$

$f = 1 \text{ Hz}$ $\qquad$ $T = 1 \text{ s}$

(e) $f = \dfrac{60 \text{ cycles}}{3600 \text{ s}}$ $\qquad$ $T = \dfrac{3600 \text{ s}}{60 \text{ cycles}}$

$f = 1.7 \times 10^{-2} \text{ Hz}$ $\qquad$ $T = 60 \text{ s}$

4. (a) This is a longitudinal vibration.
   (b) The amplitude of vibration is 3.5 cm.
   (c) In one cycle, the mass moves 4×3.5 cm = 14 cm. In 3.5 cycles, the mass moves 3.5×14 cm = 49 cm (assuming the amplitude remains constant).
5. (a) The period and frequency are reciprocals of each other. They are inversely related $\left(f = \dfrac{1}{T}\right)$.
   (b) As the length increases, the period of the pendulum increases. The period of a pendulum is directly proportional to the square root of its length.
   (c) The mass and frequency of a pendulum are independent of each other; there is no relationship between mass and frequency.
6. Moving the mass closer to the pivot point to shorten the length of the pendulum can increase the frequency. This could be demonstrated using a metronome.
7. If the force of gravity is lower, the period will be higher and the frequency will be lower $\left(T \alpha \sqrt{\dfrac{1}{g}}\right)$ and $\left(f \alpha \sqrt{g}\right)$; the clock will run slower. To increase the frequency of the pendulum to compensate, the weight would have to be moved up, which decreases the effective length of the pendulum and increases the frequency.
8. The balance wheel of a mechanical clock is a tensional pendulum (See Figure 1(c), section 6.1, page 196). The pendulum rotates back and froth with energy supplied by a spring mechanism.
9. (a) On the Moon, the gravitational force is lower than on Earth, so the period is higher $\left(T \alpha \sqrt{\dfrac{1}{g}}\right)$.
   (b) On Jupiter, the gravitational force is higher than on Earth, so the period is lower.
10. $T = 2\pi \sqrt{\dfrac{l}{g}}$

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{4\pi^2 (1.00 \text{ m})}{(2.00 \text{ s})^2}$$

$$g = 9.86 \text{ m/s}^2$$

11. butterfly stroke: in-phase arm motion
    backstroke: out-of-phase arm motion
    breaststroke: in-phase arm motion
    freestyle: out-of-phase arm motion

Applying Inquiry Skills

12. One method for determining the length of a swing without measuring it would be to measure the time for one cycle (T). Use the pendulum equation to solve for length (l).

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2}{4\pi^2}g$$

6.2 WAVE MOTION

Investigation 6.2.1 Wave Transmission: Pulses on a Coiled Spring

(Pages 203–205)

Purpose

The purpose of this investigation was to study pulses travelling along springs/ropes and their characteristics and then apply these characteristics to waves in order to understand their behaviours.

Question

(i) How do pulses move along a coiled spring?
(ii) How are pulses reflected from a fixed-end and a free-end?

Hypothesis/Prediction

(i) Pulses travel along a coiled spring through the motion of the individual coils which move back and forth as the pulse travels past a certain point.
(ii) Since the pulses are travelling along a material object, it is expected that some energy should be lost along the path due to friction. As such, the amplitude of the reflected pulse will likely be smaller than the amplitude of the original pulse, but the amplitude itself will not likely affect the speed of the pulse.
(iii) In step 8, as the tension of the spring is increased (i.e. the spring is stretched further), the speed of the pulses will increase since the spring is tighter.

Design

In this investigation, a coiled spring was stretched across the floor and subjected to various conditions (length of stretch, pulse amplitude, pulse phase, condition of end) as pulses were generated by the motion of a student's hand. The motion of the coils and behaviour of the pulses sent (speed, amplitude, phase) was observed. The following variables and controls applied in the various cases (note that part (d) was completed only if various spring types were available):

Case	Independent Variable	Dependent Variable	Controls
(a)	type of end (fixed or free)	reflected pulse phase	incident pulse phase, amplitude, spring tension
(b)	incident pulse amplitude	speed of pulse	spring tension, pulse phase, type of end
(c)	spring tension	speed of pulse	pulse amplitude, pulse phase, type of end
(d)	spring type	speed of pulse	spring tension, amplitude, pulse phase, end type

Materials

- coiled spring (Slinky toy)
- piece of paper
- various spring types (if available)
- masking tape
- stopwatch
- metre stick
- 4.0-m string